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GENERALIZED CLOSED SETS IN BINARY IDEAL TOPOLOGICAL SPACES

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Abstract. This paper deals with binary ideal topological space and discuss about generalized binary closed sets and generalized kernel in the same topological space. Further it will discuss various types of characterizations of generalized binary closed sets and generalized kernel.

1. Introduction and Preliminaries

Now a days the idea of ideal topological spaces [2] is not a new idea. So many mathematicians have worked on this field. But the notion of binary topological space is a new idea in literature and further the concept of binary ideal topological space [1] is also an another new concept in literature. The authors Al-Omari and Modak have introduced this idea. We have further considered this idea and studding some generalized closed sets and kernel and characterized them.

At first we shall consider some preliminaries idea and notions for build up this paper.

Definition 1.1. [4] Let X and Y be two nonempty sets and let $(A, B) \in \wp(X) \times \wp(Y)$ and $(C, D) \in \wp(X) \times \wp(Y)$ respectively. Then

1. $(A, B) \subseteq (C, D)$ if and only if $A \subseteq C$ and $B \subseteq D$.
2. $(A, B) = (C, D)$ if and only if $A = C$ and $B = D$.
3. $(A, B) \cup (C, D) = (H, K)$ if and only if $(A \cup C) = H$ and $(B \cup D) = K$.
4. $(A, B) \cap (C, D) = (H, K)$ if and only if $(A \cap C) = H$ and $(B \cap D) = K$.

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5. $(A, B)^c = (X \setminus A, Y \setminus B)$.
6. $(A, B) \setminus (C, D) = (A, B) \cap (C, D)^c$.

Definition 1.2. [4, 5] Let X and Y be any two non empty sets. A binary topology from X to Y is a binary structure $\mathcal{M} \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:

(i) $(\emptyset, \emptyset)$ and $(X, Y) \in \mathcal{M}$.
(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$.
(iii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_\alpha A_\alpha, \bigcup_\alpha B_\alpha) \in \mathcal{M}$.

Definition 1.3. [4, 5] If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$.

If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary space.

Proposition 1.4. [4, 5] *Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B)^{1*} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $(A, B)^{1*}$, $(A, B)^{2*}$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.*

Definition 1.5. [1] Let X and Y be any two non empty sets. A binary ideal from X to Y is a binary structure $\mathcal{I} \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:

(i) $(A, B) \in \mathcal{I}$ and $(C, D) \subseteq (A, B)$ implies $(C, D) \in \mathcal{I}$ (hereditary).
(ii) $(A_1, B_1) \in \mathcal{I}$ and $(A_2, B_2) \in \mathcal{I}$ implies $(A_1 \cup A_2, B_1 \cup B_2) \in \mathcal{I}$ (finite additivity).

Definition 1.6. [1] Let $(X, Y, \mathcal{M})$ be a binary topological space with an binary ideal $\mathcal{I}$ on $\wp(X) \times \wp(Y)$ is called ideal binary topological space and it is denoted as $(X, Y, \mathcal{M}, \mathcal{I})$. For a binary subset (A, B) of $X \times Y$, we define the following set operator: $(,)^* : \wp(X) \times \wp(Y) \to \wp(X) \times \wp(Y)$, is called a binary local function with respect to $\mathcal{M}$ and $\mathcal{I}$ is defined as follows: $(A, B)^*(\mathcal{I}, \mathcal{M}) = \{(x, y) \in (X, Y) : (U \cap A, V \cap B) \notin \mathcal{I}$ for every $(U, V) \in \mathcal{M}(x, y)\ \}$ where $\mathcal{M}(x, y) = \{(U, V) \in \mathcal{M} : (x, y) \in (U, V)\}$. In case there is no confusion $(A, B)^*(\mathcal{I}, \mathcal{M})$ is briefly denoted by $(A, B)^\star$ and is called Binary local function of (A, B) with respect to $\mathcal{I}$ and $\mathcal{M}$.

From [1], we have $C^* : \wp(X) \times \wp(Y) \to \wp(X) \times \wp(Y)$ is a Kuratowski closure operator. Therefore $\{(U, V) \subseteq (X, Y) : C^*[(X, Y) \setminus (U, V)] =$

$(X,Y)\setminus(U,V)\}$ forms a binary topology on $X\times Y$, and it is denoted as $\mathcal{M}^*$.

THEOREM 1.7. [1] *Let $(X,Y,\mathcal{M},\ \mathcal{I})$ be an ideal binary topological space. Then $\beta(\mathcal{M},\mathcal{I})=\{(V_1,V_2)\setminus I:(V_1,V_2)$ is a binary open set of $(X,Y,\mathcal{M}),I\in\mathcal{I}\}$ is a basis for $\mathcal{M}^*$.*

2. I_g-closed sets

DEFINITION 2.1. Let $(X,Y,\mathcal{M})$ be a binary topological space. Then the generalized kernel of $(A,B)\subseteq(X,Y)$ is denoted by g-ker(A,B) and defined as g-ker$(A,B)=\cap\{(U,V)\in\mathcal{M}:\ (A,B)\subseteq(U,V)\}$.

LEMMA 2.2. *Let $(X,Y,\mathcal{M})$ be a binary topological space and $(A,B)\subseteq(X,Y)$. Then g-ker$(A,B)=\{(x,y)\in X\times Y:\ b$-$Cl(\{(x,y)\})\cap(A,B)\neq(\emptyset,\emptyset)\}$.*

DEFINITION 2.3. A subset (A,B) of an ideal binary topological space $(X,Y,\mathcal{M},\mathcal{I})$ is called I_g-closed) if $(A,B)^*\subseteq(U,V)$ whenever (U,V) is binary open and $(A,B)\subseteq(U,V)$. A subset (A,B) of a binary ideal topological space $(X,Y,\mathcal{M},\mathcal{I})$ is called I_g-open if $(X,Y)\setminus(A,B)$ is I_g-closed.

THEOREM 2.4. *If $(X,Y,\mathcal{M},\mathcal{I})$ is any ideal binary topological space, then the following are equivalent:*

1. *If (A,B) is I_g-closed.*
2. *$C^*(A,B)\subseteq(U,V)$ whenever $(A,B)\subseteq(U,V)$ and (U,V) is $\mathcal{M}$-open in $X\times Y$.*
3. *$C^*(A,B)\subseteq g$-ker(A,B).*
4. *$C^*(A,B)\setminus(A,B)$ contain no nonempty $\mathcal{M}$-closed set.*
5. *$(A,B)^*\setminus(A,B)$ contains no nonempty $\mathcal{M}$-closed set.*

Proof. (1)⇒(2): If (A,B) is I_g-closed, then $(A,B)^*\subseteq(U,V)$ whenever $(A,B)\subseteq(U,V)$ and (U,V) is $\mathcal{M}$-open in $X\times Y$ and so $C^*(A,B)=(A,B)\cup(A,B)^*\subseteq(U,V)$ whenever $(A,B)\subseteq(U,V)$ and (U,V) is $\mathcal{M}$-open in $X\times Y$.

(2)⇒(3): Suppose $(x,y)\in C^*(A,B)$ and $(x,y)\notin g$-ker(A,B). Then b-$Cl(\{(x,y)\})\cap(A,B)=(\emptyset,\emptyset)$ (from Lemma 2.2). Implies that $(A,B)\subseteq(X,Y)\setminus b$-$Cl(\{(x,y)\}))$. By (2), a contradiction, since $(x,y)\in C^*(A,B)$.

(3)⇒(4): Suppose $(F,G)\subseteq C^*(A,B)\setminus(A,B)$, (F,G) is $\mathcal{M}$-closed and $(x,y)\in(F,G)$. Since $(F,G)\subseteq C^*(A,B)\setminus(A,B)$, $(F,G)\cap(A,B)=(\emptyset,\emptyset)$. We have b-$Cl(\{(x,y)\})\cap(A,B)=(\emptyset,\emptyset)$ because (F,G) is $\mathcal{M}$-closed and $(x,y)\in(F,G)$. It is a contradiction.

(4)⇒(5): This is obvious from the Definition of $C^*(A,B)$.

(5)⇒(1): Let (U,V) be a $\mathcal{M}$-open subset containing (A,B). Now $(A,B)^* \cap ((X,Y) \setminus (U,V)) \subseteq (A,B)^* \setminus (A,B)$. Since $(A,B)^*$ is a $\mathcal{M}$-closed set and intersection of two $\mathcal{M}$-closed sets is a $\mathcal{M}$-closed set, then $(A,B)^* \cap ((X,Y) \setminus (U,V))$ is a $\mathcal{M}$-closed set contained in $(A,B)^* \setminus (A,B)$. By assumption, $(A,B)^* \cap ((X,Y) \setminus (U,V)) = (\emptyset,\emptyset)$. Hence we have $(A,B)^* \subseteq (U,V)$. □

From Theorem 2.4 (3), it follows that every $\mathcal{M}$-closed is I_g-closed. Since $(F,G)^* = (\emptyset,\emptyset)$ for $(F,G) \notin \mathcal{I}$, (F,G) is I_g-closed. Since $((A,B)^*)^* \subseteq (A,B)^{b*}$, from Definition, it follows that $(A,B)^*$ is always I_g-closed for every $\mathcal{M}$-closed subset (A,B) of $X \times Y$.

Theorem 2.5. *Let $(X,Y,\mathcal{M},\mathcal{I})$ be a binary ideal topological space and $(A,B) \subseteq X \times Y$. If (A,B) is I_g-closed and $\mathcal{M}$-open then (A,B) is C^*-closed set.*

Proof. It is obvious from definition. □

Proposition 2.6. *et $(X,Y,\mathcal{M},\mathcal{I})$ be a binary ideal topological space. Let $(A,B) \subseteq X \times Y$ be I_g-closed set and (F,G) be a binary closed set, then $(A,B) \cap (F,G)$ is an I_g-closed set.*

Proof. Let $(A,B) \cap (F,G) \subseteq (U,V)$ and (U,V) is binary open set. Then $(A,B) \subseteq (U,V) \cup [(X,Y) \setminus (F,G)]$. Since (A,B) be I_g-closed we have $C^*(A,B) \subseteq (U,V) \cup [(X,Y) \setminus (F,G)]$. Now $C^*[(A,B) \cap (F,G)] \subseteq C^*(A,B) \cap (F,G) \subseteq (U,V)$. Hence $(A,B) \cap (F,G)$ is an I_g-closed. □

Definition 2.7. [6] A binary topological space $(X,Y,\mathcal{M})$ is called a binary-T_1 if for every (x_1,y_1), $(x_2,y_2) \in X \times Y$ with $x_1 \neq x_2$, $y_1 \neq y_2$, there exist (A,B), $(C,D) \in \mathcal{M}$, with $(x_1,y_1) \in (A,B)$ and $(x_2,y_2) \in (C,D)$ such that $(x_2,y_2) \in (X \setminus A, Y \setminus B)$ and $(x_1,y_1) \in (X \setminus C, Y \setminus D)$.

Proposition 2.8. [6] *The binary topological space $(X,Y,\mathcal{M})$ is binary-T_1 if and only if every binary point is binary closed.*

Theorem 2.9. *Let $(X,Y,\mathcal{M},\mathcal{I})$ be an ideal binary topological space and $(A,B) \subseteq X \times Y$. If $(X,Y,\mathcal{M})$ is a binary-T_1 space, then (A,B) is C^*-closed if and only if (A,B) is I_g-closed.*

Proof. We can complete the proof by using the Theorem 2.4 (3) and the Theorem 2.5. □

Theorem 2.10. *Let $(X,Y,\mathcal{M},\mathcal{I})$ be an ideal binary topological space and $(A,B) \subseteq X \times Y$. If (A,B) is a I_g-closed set, then the following are equivalent:*

(1) (A, B) *is a* C^**-closed set.*
(2) $C^*(A, B) \setminus (A, B)$ *is a* $\mathcal{M}$*-closed set.*
(3) $(A, B)^* \setminus (A, B)$ *is a* $\mathcal{M}$*-closed set.*

Proof. (1)⇒(2): If (A, B) is C^*-closed, then $C^*(A, B)\setminus(A, B) = (\emptyset, \emptyset)$ and so $C^*(A, B) \setminus (A, B)$ is $\mathcal{M}$-closed.

(2)⇒(3): This follows from the fact that $C^*(A, B)\setminus(A, B) = (A, B)^{b*}\setminus (A, B)$, it is clear.

(3)⇒(1): If $(A, B)^* \setminus (A, B)$ is $\mathcal{M}$-closed and (A, B) is I_g-closed, from Theorem 2.4(5), $(A, B)^{b*}\setminus(A, B) = (\emptyset, \emptyset)$ and so (A, B) is C^*-closed. □

Definition 2.11. Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space. Then the subset (A, B) of $X \times Y$ is said to be $*$-dense in itself if $(A, B)^* = (A, B)$.

Lemma 2.12. *Let* $(X, Y, \mathcal{M}, \mathcal{I})$ *be an ideal binary topological space and* $(A, B) \subseteq X \times Y$*. If* (A, B) *is* $*$*-dense in itself, then* $(A, B)^* = b\text{-}Cl((A, B)^*) = b\text{-}Cl(A, B) = C^*(A, B)$*.*

Proof. Let (A, B) be $*$-dense in itself. Then we have $(A, B) \subseteq (A, B)^*$ and hence $b - Cl(A, B) \subseteq b - Cl((A, B)^*)$. We know that $(A, B)^* = b\text{-}Cl((A, B)^*) \subseteq b\text{-}Cl(A, B)$ from Theorem 2.4 (3). In this case $b\text{-}Cl(A, B) = b\text{-}Cl((A, B)^*) = (A, B)^*$. Since $(A, B)^* = b\text{-}Cl(A, B)$, we have $C^*(A, B) = b\text{-}Cl(A, B)$. □

Definition 2.13. [7] Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space. A subset (A, B) of $X \times Y$ is called generalized binary closed if $b\text{-}Cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open in $(X, Y, \mathcal{M})$.

It is obvious that every generalized binary closed set is a I_g-closed set but not vice versa. The following Theorem shows that for $b*$-dense in itself, the concepts generalized binary closedness and I_g-closedness are equivalent.

Theorem 2.14. *If* $(X, Y, \mathcal{M}, \mathcal{I})$ *is an ideal binary topological space and* (A, B) *is* $*$*-dense in itself,* I_g*-closed subset of* $X \times Y$*, then* (A, B) *is generalized binary closed.*

Proof. Suppose (A, B) is a $b*$-dense in itself, I_g-closed subset of $X \times Y$. If (U, V) is any $\mathcal{M}$-open set containing (A, B), then by Theorem 2.4 (1), $C^*(A, B) \subset (U, V)$. Since (A, B) is $*$-dense in itself, by Lemma 2.12, $b\text{-}Cl(A, B) \subseteq (U, V)$ and so (A, B) is generalized binary closed. □

THEOREM 2.15. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space and $(A, B) \subseteq X \times Y$. Then (A, B) is I_g-closed if and only if $(A, B) = (H, G) \setminus (M, N)$ where (H, G) is C^*-closed and (M, N) contains no nonempty $\mathcal{M}$-closed set.*

Proof. If (A, B) is I_g-closed, then by Theorem 2.4 (4), $(M, N) = (A, B)^* \setminus (A, B)$ contains no nonempty $\mathcal{M}$-closed set. If $(H, G) = C^*(A, B)$, then (H, G) is C^*-closed such that $(H, G) \setminus (M, N) = ((A, B) \cup (A, B)^*) \setminus ((A, B)^* \setminus (A, B)) = ((A, B) \cup (A, B)^*) \cap (((X, Y) \setminus (A, B)^*) \cup (A, B)) = (A, B)$.

Conversely, suppose $(A, B) = (H, G) \setminus (M, N)$ where (H, G) is C^*-closed and (M, N) contains no nonempty $\mathcal{M}$-closed set. Let (U, V) be a $\mathcal{M}$-open set such that $(A, B) \subseteq (U, V)$. Then $(H, G) \setminus (M, N) \subseteq (U, V)$ which implies that $(H, G) \cap ((X, Y) \setminus (U, V)) \subseteq (M, N)$. Now $(A, B) \subseteq (H, G)$ and $(H, G)^* \subseteq (H, G)$ implies that $(A, B)^* \cap ((X \times Y) \setminus (U, V)) \subseteq (H, G)^* \cap ((X, Y) \setminus (U, V)) \subseteq (H, G) \cap ((X, Y) \setminus (U, V)) \subseteq (M, N)$. By hypothesis, since $(A, B)^* \cap ((X, Y) \setminus (U, V))$ is $\mathcal{M}$-closed, $(A, B)^* \cap ((X, Y) \setminus (U, V)) = (\emptyset, \emptyset)$ and so $(A, B)^* \subseteq (U, V)$ which implies that (A, B) is I_g-closed. □

Following Theorem gives a property of I_g-closed sets and the following Corollary follows from Theorem 2.16 and the fact that, if $(A, B) \subseteq (C, D) \subseteq (A, B)^*$, then $(A, B)^* = (C, D)^*$ and (A, B) is $*$-dense in itself.

THEOREM 2.16. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space. If (A, B) and (C, D) are subsets of $X \times Y$ such that $(A, B) \subseteq (C, D) \subseteq C^*(A, B)$ and (A, B) is I_g-closed, then (C, D) is I_g-closed.*

Proof. Since (A, B) is I_g-closed, $C^*(A, B) \setminus (A, B)$ contains no nonempty $\mathcal{M}$-closed set. Since $C^*(C, D) \setminus (C, D) \subseteq C^*(A, B) \setminus (A, B)$, $C^*(C, D) \setminus (C, D)$ contains no nonempty $\mathcal{M}$-closed set and so by Theorem 2.4 (3), (C, D) is I_g-closed. □

COROLLARY 2.17. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space. If (A, B) and (C, D) are subsets of $X \times Y$ such that $(A, B) \subseteq (C, D) \subseteq (A, B)^*$ and (A, B) is I_g-closed, then (A, B) and (C, D) is generalized binary closed.*

THEOREM 2.18. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space and $(A, B) \subseteq X \times Y$. Then (A, B) is I_g-open if and only if $(F, G) \subseteq Int^*(A, B)$ whenever (F, G) is $\mathcal{M}$-closed and $(F, G) \subseteq (A, B)$ (where Int^* denotes the interior operator of $\mathcal{M}^*$-topology on $X \times Y$).*

Proof. Suppose (A, B) is I_g-open. If (F, G) is $\mathcal{M}$-closed and $(F, G) \subseteq (A, B)$, then $(X, Y) \setminus (A, B) \subseteq (X, Y) \setminus (F, G)$ and so $C^*((X, Y) \setminus (A, B)) \subseteq ((X, Y) \setminus (F, G)$. Therefore, $(F, G) \subseteq Int^*(A, B)$.

Conversely, suppose the condition holds. Let (U, V) be a $\mathcal{M}$-open set such that $(X, Y) \setminus (A, B) \subseteq (U, V)$. Then $(X, Y) \setminus (U, V) \subseteq (A, B)$ and so $(X, Y) \setminus (U, V) \subseteq Int^*(A, B)$ which implies that $C^*((X, Y) \setminus (A, B)) \subseteq (U, V)$. Therefore, $(X, Y) \setminus (A, B)$ is I_g-closed and so (A, B) is I_g-open. □

THEOREM 2.19. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space and $(A, B) \subseteq X \times Y$. If (A, B) is I_g-open and $Int^*(A, B) \subseteq (C, D) \subseteq (A, B)$, then (C, D) is I_g-open.*

Proof. Proof is obvious from above theorem. □

The following theorem gives a characterization of I_g-closed sets in terms of I_g-open sets.

THEOREM 2.20. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space and $(A, B) \subseteq X \times Y$. Then following are equivalent:*

(1) *(A, B) is I_g-closed.*
(2) *$(A, B) \cup ((X \times Y) \setminus (A, B)^*)$ is I_g-closed.*
(3) *$(A, B)^* \setminus (A, B)$ is I_g-open.*

Proof. (1)⇒(2): Suppose (A, B) is I_g-closed. If (U, V) is any $\mathcal{M}$-open set such that $((A, B) \cup ((X, Y) \setminus (A, B)^*)) \subseteq (U, V)$, then $((X, Y) \setminus (U, V)) \subseteq (X, Y) \setminus [(A, B) \cup ((X, Y) \setminus (A, B)^*)] = (A, B) \setminus (A, B)^*$. Since (A, B) is I_g-closed, by Theorem 2.4(4), it follows that $(X, Y) \setminus (U, V) = (\emptyset, \emptyset)$ and so $(X, Y) = (U, V)$. Since (X, Y) is only $\mathcal{M}$-open set containing $(A, B) \cup ((X, Y) \setminus (A, B)^*)$, clearly, $(A, B) \cup ((X, Y) \setminus (A, B)^*)$ is I_g-closed.

(2)⇒(1): Suppose $(A, B) \cup ((X, Y) \setminus (A, B)^*)$ is I_g-closed. If (F, G) is any $\mathcal{M}$-closed set such that $(F, G) \subseteq (A, B)^* \setminus (A, B)$, then $(A, B) \cup ((X, Y) \setminus (A, B)^*) \subseteq (X, Y) \setminus (F, G)$ and $(X, Y) \setminus (F, G)$ is $\mathcal{M}$-open. Therefore, $[(A, B) \cup ((X, Y) \setminus (A, B)^*)]^* \subseteq (X, Y) \setminus (F, G)$ which implies that $(A, B)^* \cup ((X, Y) \setminus (A, B)^*)^* \subseteq (X, Y) \setminus (F, G)$ and so $(F, G) \subseteq (X, Y) \setminus (A, B)$. Since $(F, G) \subseteq (A, B)^*$, it follows that $(F, G) = (\emptyset, \emptyset)$. Hence (A, B) is I_g-closed.

The equivalence of (2) and (3) follows from the fact that $(X, Y) \setminus ((A, B)^* \setminus (A, B) = (A, B) \cup ((X, Y) \setminus (A, B)^*)$. □

THEOREM 2.21. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space. Then every subset of $X \times Y$ is I_g-closed if and only if every $\mathcal{M}$-open set is C^*-closed.*

Proof. Suppose every subset of $X \times Y$ is I_g-closed. If (U, V) is $\mathcal{M}$-open, then (U, V) is I_g-closed and so $(U, V)^* \subseteq (U, V)$. Hence (U, V) is C^*-closed.

Conversely, suppose that every $\mathcal{M}$-open set is C^*-closed. If $(A, B) \subseteq X \times Y$ and (U, V) is a $\mathcal{M}$-open set such that $(A, B) \subseteq (U, V)$, then $(A, B)^* \subseteq (U, V)^* \subseteq (U, V)$ and so (A, B) is I_g-closed. □

Definition 2.22. An ideal binary topological space $(X, Y, \mathcal{M}, \mathcal{I})$ is called an $T_{\mathcal{I}}$-space if every I_g-closed is $\mathcal{M}^*$-closed.

Theorem 2.23. *Let $(X, Y, \mathcal{M}, \mathcal{I})$ be an ideal binary topological space, the following conditions are equivalent:*

(1) *(X, Y) is a binary $T_{\mathcal{I}}$-space.*
(2) *Every point $(x, y) \in (X, Y, \mathcal{M}, \mathcal{I})$ is either binary closed or $\mathcal{M}^*$-open.*

Proof. (1) $\Rightarrow$ (2): Let $(x, y) \in (X, Y)$. If $\{(x, y)\}$ is not binary closed, then $(A, B) = (X, Y) \setminus \{(x, y)\} \notin \mathcal{M}$ and then (A, B) is trivially I_g-closed. By (1), (A, B) is $\mathcal{M}^*$-closed. Hence $\{(x, y)\}$ is $\mathcal{M}^*$-open.
(2) $\Rightarrow$ (1): Let (A, B) be I_g-closed and let $(x, y) \in C^*(A, B)$. We have the following two cases:
Case 1. $\{(x, y)\}$ is binary closed. By Theorem 2.4, $(A, B)^* \setminus (A, B)$ dose not contain a nonempty binary closed subset. This show that $(x, y) \in (A, B)$.
Case 2. $\{(x, y)\}$ is $\mathcal{M}^*$-open. Then $\{(x, y)\} \cap (A, B) \neq \emptyset$. Hence $(x, y) \in (A, B)$.
Thus in both cases $(x, y) \in (A, B)$ and so $C^*(A, B) = (A, B)$. Hence (A, B) is $\mathcal{M}^*$-closed, which show that (X, Y) is a binary $T_{\mathcal{I}}$-space. □

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